This may be written

$$\frac{\frac{d}{dx}(px-y)}{px-y}=-\frac{1}{y}\frac{dy}{dx},$$

the integral of which is $pxy-y^2=c$.

Eliminating p between this and the given equation,

$$y^2+\frac{c}{c-h^2}=-c.$$

Putting $\frac{c}{c-h^2}=-c'$, $y^2-c'x^2=\frac{c'h^2}{1+c'}$.

This solution may also be obtained by solving $pxy-y^2=c$, obtaining $y^2+c'x^2=c$.

This gives $p=-\frac{c'x}{y}$, which substituted in the given equation makes $c=\frac{-c'h^2}{1+c'}$.

Also solved by the *PROPOSER*.

MECHANICS.

79. **Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.**

The four wheels of a street car are rigidly fixed to their axles so that axles and wheels turn together. Is it more advantageous to apply the brakes to the front or to the rear wheels, supposing the brakes to block the wheels in each case?

Solution by the PROPOSER.

The question may be answered by treating the problem as a statical one, thus: Suppose the car placed upon an inclined plane and let us inquire in which case may the plane be raised to the greater angle before slipping begins. Let $2a$ be the distance between the centers of the wheels, each of radius c, b the distance of the center of gravity, G of the car above (or below) this line of centers, w the weight of each of the trucks, w_1 the weight of car.

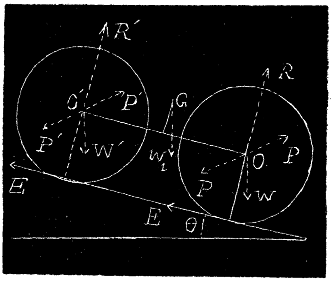
Take the case first when brakes are applied to rear wheels, there being in this case, of course, no friction between the front wheels and plane. Consider the figure as consisting of two rigid bodies, viz, the front trucks, and the car with the rear trucks. The forces acting upon the front trucks are their weight w, the reaction R of the plane, and a force, P, at O obliquely downward, which is the resultant of w and the backward pull of the car and rear wheels. Upon the car at this point O there will also be an equal and opposite force to P, the resultant of R and w.

The forces on car and rear wheels are, this force P at O, w_1 at G, w at O', R_1 the reaction of the plane, and F the force of friction.

From forces on front trucks we have,

$$\left.\begin{array}{l}(1).\ w\sin\theta=P\cos\psi \\ (2).\ w\cos\theta=R-P\sin\psi\end{array}\right\}$$ ψ being the angle between P and the line of centers OO'.

From forces on car and rear wheels, letting μ be coefficient of friction, taking moments about O' and resolving horizontally and vertically,



(3). $2aP\sin\psi=w_1(a+b\tan\theta)\cos\theta-\mu R_1c$.

(4). $\mu R_1=(w+w_1)\sin\theta+P\cos\psi$.

(5). $R_1+P\sin\psi=(w+w_1)\cos\theta$.

From these five equations we get,

$$\tan\theta=\frac{a\mu(2w+w_1)}{aw_1b+(2a-c\mu)(2w+w_1)}\ldots\ldots(A).$$

Next take the case when brakes are applied to front wheels, considering in this case, the rear trucks as one rigid body, and the car with front trucks as the other. The two oblique forces P now act at O' one upon the car and one upon rear wheels (lettered P' to avoid confusion).

From forces on rear wheels we get,

$$\left.\begin{array}{l}(a).\ w\sin\theta=P'\cos\psi' \\ (b).\ w\cos\theta=R_1-P'\sin\psi'\end{array}\right\}$$ ψ' being the angle between P' and OO'.

From forces on car and front wheels, taking moments about O, resolving horizontally and vertically.

(c). $2aP'\sin\psi'=w_1(a-b\tan\theta)\cos\theta-\mu Rc$.

(d). $\mu R=(w+w_1)\sin\theta+P'\cos\psi'$.

(e). $R+P'\sin\psi'=(w+w_1)\cos\theta$.

From these five equations we get

$$\tan\theta=\frac{a\mu(2w+w_1)}{(2a-\mu c)(2w+w_1)-\mu w_1b}\ldots\ldots(B).$$

Comparing (B) with (A) we see that in the latter case $\tan\theta$, and hence θ, is the greater, and hence we infer it is more advantageous to apply the brakes to the front wheels.

80. Proposed by B. F. FINKEL, A. M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A circular board is placed on a smooth horizontal plane and a boy runs with uniform speed around on the board close to the edge. Find the motion of the center of the board.